

Assessing Students' Conviction in Writing Mathematical Proofs

I. Maryono^{1,2}, R. Kariadinata¹, T. K. Rachmawati¹ and I. Nuraida¹

¹Mathematics Program, UIN Sunan Gunung Djati Bandung, Jl. A.H. Nasution No. 105, Bandung, Indonesia

²School of Postgraduate Studies, Universitas Pendidikan Indonesia, Jl. Setiabudi No. 225, Bandung, Indonesia
{iyonmaryono, rahayu.kariadinata, tikakarlinarachmawati, idanuraida}@uinsgd.ac.id

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Abstract: Assessing self-conviction in writing mathematical proofs is important to provide feedback for students. The purpose of this study is to determine the correlation between the levels of self-conviction with the score of writing evidence and to provide feedback for students who are still wrong in writing the proofs. The method used in this study is a correlation study between the levels of self-conviction to write the proofs with the score of writing the proofs. The subjects of this study were forty third semester students of mathematics education. These participants were tested writing proofs along with the claim, then the correlation between the claim with the score of the write proofs was examined. Furthermore, students who believe that their proofs are wrong were interviewed. The results of this study indicated that the correlation between the level of confidence with the score of the ability to write mathematical proofs was weak. Despite their weak correlation, assessment of conviction levels is important for providing feedback to students who believe in the proofs they have written even though the proofs they have written were still wrong. The results of this study imply that mathematics learning that focuses on the ability to provide mathematical proofs must provide an assessment on the aspects of student self-conviction.

1 INTRODUCTION

Assessing students' self-conviction in writing mathematical evidence is important. The results of this assessment can be used as feedback for students regarding their understanding. In writing the evidence, convincing oneself is the first level before convincing others. Mason, Burton and Stacey (2010) states that there are three levels in the convincing process: (1) convince yourself; (2) convince a friend; (3) convince an enemy.

One of the functions of mathematical proofs and the action of making proofs is to verify and justify a proposition (Bell, 1976; Renz, 1981; Villiers, 1990). The activity of verification and justification cannot be separated from the self-conviction level of the claim. Therefore, the degree of self-conviction in claims is a factor to be considered in providing an assessment of the claim.

The facts show that some students have the view that convincing arguments are different from mathematical proofs. They are convinced of his opinion even though that opinion is not a mathematical proof (Weber, 2010). This shows that self-conviction in the claim of proof does not guarantee that the claim is true. In some cases

students sometimes do not believe in the truth of mathematical proofs. They are more convinced in the truth of their inductively obtained claims. Therefore, the lecturer needs to provide an assessment (feedback) on the student's claim to differ which claim is a mathematical proof, and where is not a mathematical proof.

The main purpose of this study is to investigate the correlation of students' self-conviction in writing the proofs with the truth of the proofs they write and to trace the causes in the case of students who have high self-conviction but low in writing proofs. The researcher gave the test of writing mathematical proofs to 40 students after attending a lecture for seven meetings with self-explanation technique. At each step of proof-writing, the student must provide his or her claims in two choices that are "certain" and "less certain" with the assumption that the "less certain" option is indicated by not providing an answer.

The results of the study provide a description on how the relationship of student self-conviction to his claim. Furthermore, feedback to students may be given in three possibilities, namely: 'sure and true' is a category of students who have high confidence and score. 'not sure but true' is a category of students who have low self-conviction but having high scores. 'sure

but not true' is a category of students who have high confidence but low scores.

2 LITERATURE REVIEW

The essence of the assessment is to look for a link between what should be learned in the curriculum and what students have learned. So the main problem in the assessment is how to recognize the existing learning and data about what has been learned by students (Cumming and Wyatt-Smith, 2009). Therefore, the assessment should pay attention to the learning process and learning achievement so as to obtain accurate conclusions about the condition of the students.

Students' self-conviction in the claim of writing mathematical proofs needs to be revealed so that feedback can be provided for students who are very convinced that their claims are true, but the claims are actually wrong. This is where the importance of assessment, which serves as a means to obtain information about students' knowledge, motivation and potential and to provide feedback (Latta, 2007 and Ginsburg, 2009).

In mathematics, proving is the method used to derive a clear conclusion. The importance of the ability for mathematics teacher candidates to give proofs can be described in the function of proofs and proving acts, such as : (1) verification or justification on a proposition; (2) explanation to the truth; (3) systematisation; (4) discovery of new findings; (5) communication (Bell, 1976 and Villiers, 1990). The importance of the proofs can also be reviewed based on the purpose. Renz (1981) describes seven objectives of evidence in mathematics, namely to: (1) Clarify the relationships between traits; (2) Giving us pleasure in constructing arguments and finding out the proof; (3) Helps remember important and useful results; (4) Guiding us along the right path formally where our intuition may be weak or misleading; (5) Guiding calculations; (6) Exploring the nature of the formal system; (7) Offering a different perspective.

From a pedagogical point of view, proving is a process of convincing the validity of a statement through logical arguments. There are three levels in the convincing process: (1) convince yourself; (2) convince a friend; (3) convince an enemy (Mason, Burton and Stacey; 2010). In the process of self-conviction, one should be convinced to oneself. However, self-conviction in the truth of the written argument does not guarantee that the argument is valid.

The process of learning to practice the ability to prove at least consists of: (1) providing counterexamples to claims that are false; (2)

evaluating a statement to know its truth by justification; (3) analysing the work of another student whether there is still a mistake in his reasoning (Thompson, 2012). Technique used in this research is self-explanation technique. This technique provides guidance to students in learning proof by asking questions: (1) Do you understand the idea? (2) Do you understand why the idea is used?, (3) How can the idea be used/linked to other ideas (other theorems, prior knowledge) in proof? (Hodds, Alcock, and Inglis, 2014)..

3 METHODS

The method used in this study was a correlation study between the level of confidence with the truth of writing proofs. The subjects of this research were 40 students of mathematics teacher candidates in third semester. The instruments used in this study were proving ability test and interview guidance.

This research tries to analyse the results of proof writing skill test from 40 students of mathematics teacher candidate. In the test instructions the students were instructed to write their conviction on each proving steps in two categories: Sure and Less Sure. The first data obtained is the scores of writing proofs and the level of self-conviction. These two data were tested for their correlation resulting in several categories of students, namely: 'sure and true'; 'Sure but not true'; and 'less sure but true'. Furthermore, researchers interviewed students who categorized 'sure but not true'. The level of truth consists of two categories namely high and low. The level is high (ranging from 70 to 100) and low (ranging from 0 to 60). Level of conviction is divided into two categories, namely high and low. The high category is in the range of 70% to 80% and the low category ranges from 00% to 60% of the standard proof measures performed.

4 RESULTS

The correlation analysis between the conviction levels with the ability to write proofs is presented in Table 1. This table shows there is a positive weak correlation between the levels of conviction with the score of proof writing ability at 0.361. This means that the relationship between the level of confidence with the score of writing ability was linear, indicating that the higher the level of conviction, the higher the acquired score of the ability to write proofs. Similarly, the lower the level of conviction, the lower the acquired score of the ability to write proofs.

Table 1: Correlation between the conviction levels with the ability to write proofs

		Score	Level of conviction
Score	Pearson Correlation	1	0.36*
	Sig. (2-tailed)		0.02
	N	40	40

*. Correlation is significant at the 0.05 level (2-tailed).

Furthermore, the hypothesis with the level of trust $\alpha = 0.05$ was tested and the conclusion is there is a significant relationship between the level of conviction to the scores of proof writing ability. Based on the coefficient of determination, the level of confidence to the score of proof writing ability was 13.0321% and 86.9679% was determined by other variables..

Table 2 shows that five students have very high confidence when the truth score of writing evidence is very low. This usually causes students to be disappointed because their expectations are far from reality.

Table 2: The scores of students

Name	Level of conviction (%)	Scores (%)
A	76.8	25
B	86	25
C	100	45
D	100	50
E	100	50

Based on the interviews of these five students, it is found that (1) they do not understand the axiom system; (2) they cannot connect between concepts in an axiomatic system; (3) they were in rush to make conclusions, with lack of reasoning to be declared right.

5 DISCUSSION

In the context of writing mathematical proof, the factor of self-conviction is not enough to guarantee the truth of proofs. What constitutes a convincing argument for one person may not at all convince others (Harel and Sowder, 1998). This is consistent with the findings in this study that the self-confidence level of self-confidence score was only 13.0321%. However, since the standard of evidence in mathematics is clear, the role of the assessment of mathematical proof claims is crucial to the success of the student.

There are two components in the assessment of mathematical proofs that are self-understanding of

the principles of proof and the ability to write the proofs (McCrone and Martin, 2004). In this study the assessment of mathematical proofs was done on the ability to write proofs and conviction against self-claims. Based on the assessment results of the claim to write proofs, the lecturer can provide feedback for students who believe the proof is true and still wrong. The findings in this study as feedback for students are (1) understanding the axiom system so that it caught what became the relationship between concepts; (2) the student must re-examine whether the causal link of the chain of statements made logical or not, whether there is still a disconnected or not.

6 CONCLUSION

There is a weak correlation between the level of conviction with the score of the ability to write mathematical proofs. Although there was a weak influence of self-confidence on the truth of this mathematical proof, the assessment of the level of self-conviction is useful to provide feedback for students who believe in their claims, but found false in proving.

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