Modified Fourth-Order Runge-Kutta Method Based on Trapezoid Approach

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Abstract—**This paper analyzes the modification of fourth order Runge-Kutta Method based on Trapezoid approach for calculating a distance of accelerometer sensor. This modification is done to obtain the fourth-order Runge-Kutta method to calculate a more accurate and precise position. In this research, we compared the result between original fourth-order Runge-Kutta Method and modified fourth-order Runge-Kutta Method. We also compared the accuracy and precision of those methods. The result shows that modified fourth-order Runge-Kutta Method has a better accuracy and precision. Original fourth-order Runge-Kutta Method has an average inaccuracy value of 4.93%, and modified fourth-order Runge-Kutta Method has an average inaccuracy value of 4.11%. Meanwhile, for the precision level, original fourth-order Runge-Kutta Method has average imprecision value of 32.52%, and modified fourth-order Runge-Kutta Method has average imprecision value of 29.66%.**

Keywords—modified fourth-order Runge-Kutta; Trapezoid approach; Accelerometer sensor

I. INTRODUCTION

Over the last ten years, there have been a lot of devices that can measure distance. One of the best devices was GPS, but GPS systems are useless when GPS is used in closed spaces because GPS has a permanent communication with satellites. Accelerometer sensor can be an alternate device that we can use to measure distance in a closed space[1]. Measuring distance using accelerometer sensor can be done by integrating twice the acceleration data, but the errors are accumulating much faster than expected[1]. Choosing an integration method is necessary for solving the problem. One of the integration method that can be used is a fourth-order Runge-Kutta Method[2][3][4].

A fourth-order Runge-Kutta Method could be used for analyzing the graphical model. The Reference[5] analyzed the Rosen Zweig-Mac Arthur model, by comparing fourth-order Runge-Kutta and Adam-Moultan. The method can also be used for quantifying a flow of water molecules by combining fourth-order Runge-Kutta and Euler for Tractography in Diffusion-Weighted Magnetic Resonance Imaging[6].

Previous studies into modified fourth-order Runge-Kutta has been done by using several ways. The reference [7]was modified fourth-order Runge-Kutta based on arithmetic and geometric means. In another research, fourth-order Runge-Kutta was modified by using the Heronian mean formula[8]. Trapezoid rule can be applied for generating symplectic Runge-Kutta Method[9].

Accelerometer sensor can be applied in telecommunication field. In a positioning system, accelerometer sensor is combined with Real-Time Kinematic Global Positioning System (RTK-GPS) for measuring displacement to get an optimal result[10]. Accelerometer sensor can also be applied for determining a position in a smartphone to measure walking step by combining accelerometer sensor and GPS [11]. In the marine sector, Accelerometer sensor is used in antenna stabilization for satellite tracking [12][13].

In this research modified fourth-order, Runge-Kutta Method based on Trapezoid approach was aiming to get a better accuracy and precision a fourth-order Runge-Kutta Method. Application of this modified integration method for calculating distance using accelerometer is being a novelty value in this research.

II. RESEARCH METHOD

Fig.1 shows the research method that has five steps in this research.

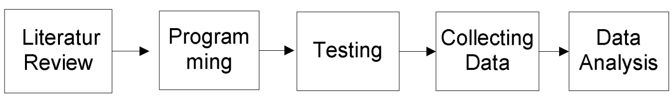

Fig. 1. Research Method

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In this research, we use microcontroller 8 bit for programming for reading and processing accelerometer data. We use an inertial measurement unit (IMU) Gy-88 to get the accelerometer data. The sensor has three kinds of integrated circuit (IC) that is MPU-6000A[14], BMP085[14], BMP085[15], dan HMC5883L[16]. Fig 2 below shows the flowchart algorithm.

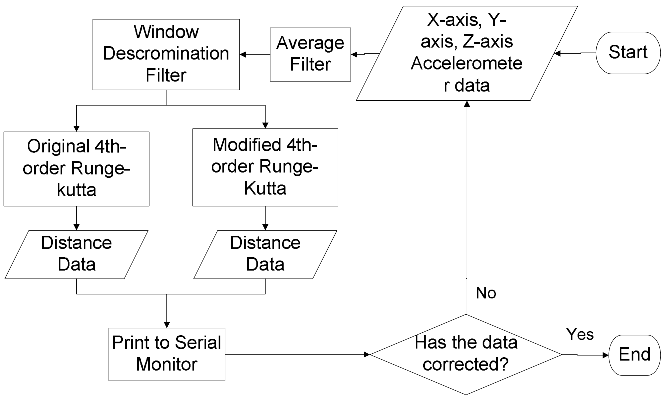

Fig. 2. Flowchart Algorithm

Fig 2 explains about processing data from accelerometer sensor using two types filter, that is average filter and window discrimination filter alternately. Filtered data is used for measuring distance by integrating the data using two methods, that is: a) an original fourth-order Runge-Kutta method and b) modified a fourth-order Runge-Kutta based on Trapezoid approach. The distance is displayed on serial monitor.

Measuring distance using accelerometer sensor can be done by integrating twice the acceleration data, but the errors are accumulating much faster than expected. We need Average Filter to eliminate the errors. Average Filter operates by averaging the input data to get the new output. Equation (1) was the mathematical equation of average[17][18].

$$a_{n+1} = \frac{1}{n}\sum_{n=1} a_n \quad (1)$$

where$a_n$is acceleration data from the accelerometer.

The reality is the noise keeps being detected when the accelerometer sensor is motionless because is there another noise left in the accelerometer sensor. We cannot ignore the noise when integration is being processed, because the accumulating noise makes the results are not satisfactory. It needs discrimination window filter for eliminating the noise. Discrimination window filter is eliminating data that out of range[19]. Based on the measurement, we used range value of -0.1 m / s2 to 0.1 m / s2.

In this research, we compared the result distance between original fourth-order Runge-Kutta dan modified fourth-order Runge-Kutta. The aim was to get a better accuracy and precision fourth-order Runge-Kutta Method for measuring distance using an accelerometer sensor. We hope this modified a fourth order Runge-Kutta based on Trapezoid approach can be better to eliminate the noise accumulation.

In detail information of fourth-order, Runge-Kutta Method and modified a fourth-order Runge-Kutta Method can be seen below.

A. Original a Fourth-Order Runge-Kutta

Original a fourth-order Runge-Kutta Method is more accurate than the lower order ones and hence it is the most popular one. Equation (2) to (6) is the formula of an original fourth-order Runge-Kutta Method[5].

$$k_1 = f(x_i, y_i) \quad (2)$$

$$k_2 = f(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1 h) \quad (3)$$

$$k_3 = f(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2 h) \quad (4)$$

$$k_4 = f(x_i + h, y_i + k_3) \quad (5)$$

$$y_{i+1} = y_i + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h \quad (6)$$

Wherey_{i+1} is the final result,$f(x_i, y_i)$is the first derivative of y_i, h is the sampling time, k_1 is a slope increment at the beginning of the interval, k_2 and k_3 are increments based on the slope at the interval center point, and $k_4$is the increase based on the slope at the end of the interval.

B. Modified a Fourth-order Runge-Kutta Based on Trapezoid Approach

Modified a fourth-order Runge-Kutta based on Trapezoid approach was aiming to get a better accuracy and precision for measuring distance using accelerometer sensor. Equation (7) is the formula of Trapezoid rule.

$$y_{i+1} = y_i + \frac{f(x_i, y_i) + f(x_{i-1}, y_{i-1})}{2} h \quad (7)$$

Equation (8) to (12) is the formula of modified a fourth-order Runge-Kutta based on Trapezoid approach.

$$k_1 = \frac{f(x_i, y_i) + f(x_{i-1}, y_{i-1})}{2} \quad (8)$$

$$k_2 = \frac{f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right) + f(x_i - \frac{1}{2}h, y_{i-1} + \frac{1}{2}k_1 h)}{2} \quad (9)$$

$$k_3 = \frac{f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h\right) + f(x_i - \frac{1}{2}h, y_{i-1} + \frac{1}{2}k_2 h)}{2} \quad (10)$$

$$k_4 = \frac{f(x_i+h,\ y_i+k_3)+f(x_i,y_i+k_3)}{2} \tag{11}$$

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h \tag{12}$$

In this research, we use a linear actuator table for testing the accelerometer sensor. Fig 3 shows the experimental setup in this research. We have six distance variant for being parameters of this research that is: 5 cm, 10 cm, 15 cm, 20 cm, 25 cm, and 30 cm.

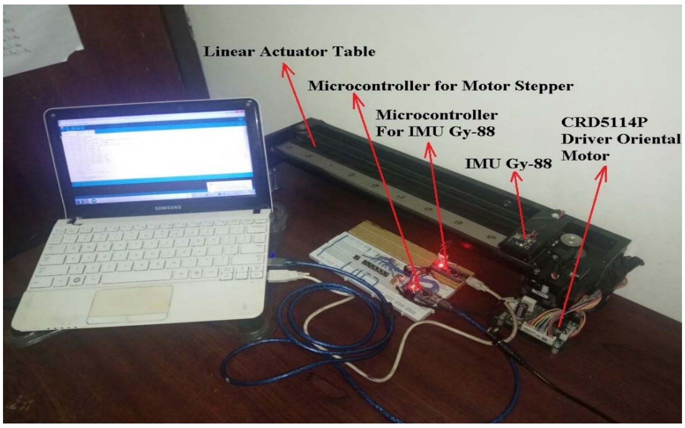

Fig. 3. Experimental Setup

Fig. 3 shows the experimental setup of this research. We put accelerometer sensor at the linear actuator table and measuring acceleration data when the linear actuator table is moving. We use average speed of 0.02 m/s. Linear actuator table is moving based on pulse generating from the microcontroller. We need the CRD5114P driver for converting pulse to the motor stepper.

III. Result and Analysis

Based on testing by using an experimental setup, we get a result that is distance data in table form. Table I shows a result of integration using original fourth-order Runge-Kutta, and Table II shows a result of integration using modified a fourth-order Runge-Kutta.

TABLE I. Original Fourth-Order Runge-Kutta

Trial to-	Distance (cm) 5	10	15	20	25	30
1	5.57	9.63	15.59	20.73	25.43	30.73
2	5.48	10.95	15.72	21.49	26.38	30.22
3	5.55	11.06	15.86	21.22	25.64	30.63
4	5.25	10.24	15.63	21.10	25.21	30.91
5	5.22	10.37	16.00	21.82	25.87	30.95
6	5.29	10.96	16.03	21.23	25.59	30.22
7	5.30	10.30	15.90	20.42	25.09	31.20
Average	5.38	10.50	15.82	21.14	25.60	30.69
Deviation standard	0.14	0.48	0.16	0.43	0.40	0.34

TABLE II. Modified Fourth-Order Runge-Kutta

Trial to-	Distance (cm) 5	10	15	20	25	30
1	5.42	9.61	15.57	20.65	25.30	30.72
2	5.33	10.93	15.70	21.28	26.25	30.21
3	5.33	10.92	15.85	21.00	25.63	30.60
4	5.11	10.19	15.57	20.88	25.14	30.90
5	5.12	10.35	15.91	21.31	25.74	30.94
6	5.18	10.82	16.01	21.15	25.52	30.21
7	5.23	10.28	15.81	20.30	25.01	31.19
Average	5.25	10.44	15.77	20.94	25.51	30.68
Deviation standard	0.11	0.45	0.16	0.34	0.39	0.34

Based on Table I and II, the distance measuring by using an original fourth-order Runge-Kutta Method has average value of 5.38 for the real distance of 5 cm, 10.50 for the real distance of 10 cm, 15.82 for the real distance of 15 cm, 21.14 for the real distance of 20 cm, 25.60 for the real distance of 25 cm, and 30.69 for the real distance of 30 cm. The distance measuring by using modified fourth-order Runge-Kutta based on Trapezoid approach has average value of 5.25 for the real distance of 5 cm, 10.44 for the real distance of 10 cm, 15.77 for the real distance of 15 cm, 20.94 for the real distance of 20 cm, 25.51 for the real distance of 25 cm, and 30. for the real distance of 30 cm.

The result shows that modified a fourth-order Runge-Kutta is better than an original fourth-order Runge-Kutta. But, we have to see how good accuracy of those methods. Accuracy is the degree of closeness of measurements of a quantity to that quantity is true value[20]. Inaccuracy value is affected by noise while the data is integrating. To know how good accuracy of those methods, we need to calculate inaccuracy that is the difference between measured value and known value. Fig. 4 shows the percentage of the average inaccuracy value of those methods.

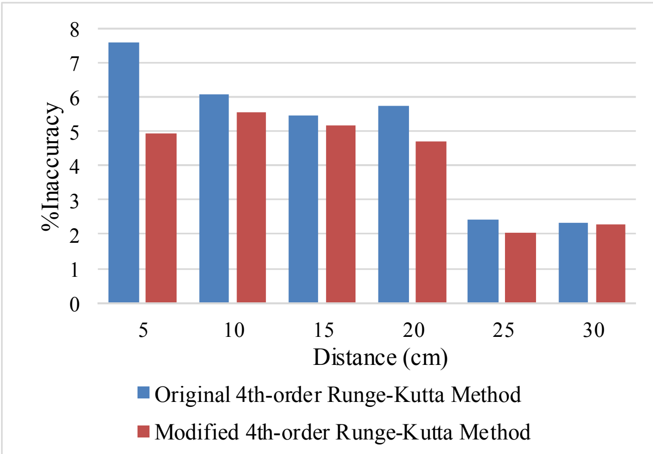

Fig. 4. InaccuracyPercentage

Based on Fig. 4, an original fourth-order Runge-Kutta has inaccuracy value of 7.60% for the real distance of 5 cm, 6.07% for the real distance of 10 cm, 5.46% for the real distance of 15 cm, 5.72% for the real distance of 20 cm, 2.41 % for the real distance of 25 cm, 2.31% for the real distance of 30 cm, and modified a fourth-order Runge-Kutta Method has inaccuracy value of 4.91 % for the real distance of 5 cm, 5.54% for the real distance of 10 cm, 5.16% for the real distance of 15 cm, 4.69% for the real distance of 20 cm, 2.05% for the real distance of 25 cm, 2.27% for the real distance of 30 cm. The result shows that modified a fourth-order Runge-Kutta Method has bigger inaccuracy value than original fourth-order Runge-Kutta Method. The original method has an average inaccuracy value of 4.93%, and modified method has an average inaccuracy value of 4.11%.

We also need to know the precision of those methods for knowing how stable the methods. Precision is the degree to which repeated measurements under unchanged conditions show the same results[20]. To know how good precision of those methods, we need to calculate imprecision value. Fig. 5 shows the percentage of average imprecision value of those methods.

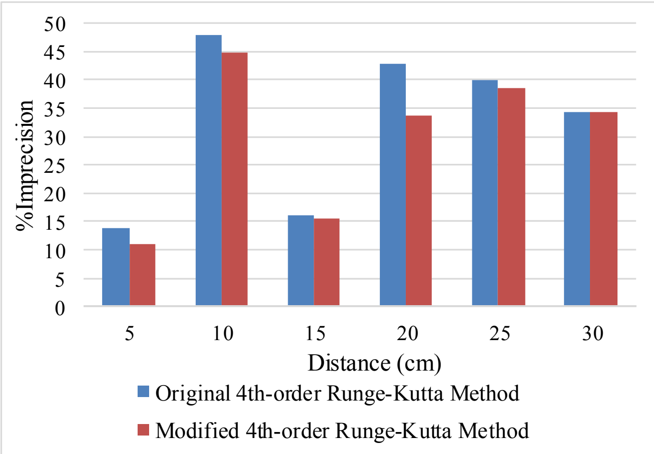

Fig. 5. Imprecision Percentage

Based on Fig. 5, original fourth-order Runge-Kutta has imprecision value of 13.73% for the real distance of 5 cm, 47.94% for the real distance of 10 cm, 16.19% for the real distance of 15 cm, 42.88% for the real distance of 20 cm, 40.07% for the real distance of 25 cm, 34.29% for the real distance of 30 cm, and modified fourth order Runge-Kutta is 10.91% for the real distance of 5 cm, 44.72% for the real distance of 10 cm, 15.58% for the real distance of 15 cm, 33.75% for the real distance of 20 cm, 38.65% for the real distance of 25 cm, 34.34% for the real distance of 30 cm. The result shows that modified fourth-order Runge-Kutta Method has bigger imprecision value than the original fourth-order Runge-Kutta Method. The original method has an average imprecision value of 32.52%, and modified method has an average imprecision value of 29.66%.

IV. Conclusion

This study was conducted based on a modification of the fourth-order Runge-Kutta method to calculate the distance using the accelerometer sensor, then compare the distance measurement results between the original method and modified method which is modified. The accelerated data is filtered using a filter average and a descrimnation filter window in sequence. The descrimnation filter window requires an accelerometer value when the accelerometer is not moving. Based on the test results of an accelerometer sensor noise at rest, gained the upper and lower thresholds for descrimnation window filter are -0.1 m / s2 and 0.1 m / s2.

Modified a fourth-order Runge-Kutta Method has a better accuracy and precision. Original fourth-order Runge-Kutta Method has an average inaccuracy value of 4.93% and modified a fourth-order Runge-Kutta Method has an average value of 4.11%. Meanwhile, for the precision level, an original fourth-order Runge-Kutta Method has an average imprecision value of 32.52% and modified a fourth-order Runge-Kutta Method has an average imprecision value of 29.66%.

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