ABSTRAK

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Judul : Karakterisasi Graf Annihilator Ideal Dari Ring Z_n

Misal R adalah ring komutatif dengan identitas bukan nol dan I ideal sejati di R, graf annihilator R sehubungan dengan I dinotasikan dengan $AG_{\mathrm{I}}(R)$, yaitu sebuah graf tak berarah dengan himpunan titiknya $V\left(AG_{\mathrm{I}}(R)\right) = \{x \in R \backslash I;\ xy \in I$ untuk suatu $y \notin I\}$, dan dua titik berbeda x dan y bertetangga jika dan hanya jika $A_{\mathrm{I}}(xy) \neq A_{\mathrm{I}}(x) \cup A_{\mathrm{I}}(y)$, dimana $A_{\mathrm{I}}(x) = \{r \in R: rx \in I\}$. Kemudian $AG_{\mathrm{I}}(R)$ dikembangkan dengan ring khusus (Z_{n}) dimana (Z_{n}) adalah ring komutatif dari bilangan bulat modulo *n* dengan ideal sejati I di (Z_{n}) dinotasikan dengan $AG_{\mathrm{I}}(Z_{\mathrm{n}})$ adalah sebuah graf tak berarah dengan himpunan titiknya $V\left(AG_{\mathrm{I}}(Z_{\mathrm{n}})\right) = \{x \in Z_{\mathrm{n}} \backslash I;\ xy \in I, y \notin I\}$, dan dua titik berbeda x dan y bertetangga jika dan hanya jika $A_{\mathrm{I}}(xy) \neq A_{\mathrm{I}}(x) A_{\mathrm{I}}(y)$, dimana $A_{\mathrm{I}}(x) = \{r \in Z_{\mathrm{n}}: rx \in I\}$ untuk suatu $x \in Z_{\mathrm{n}}$. Graf tak berarah $\Gamma_{\mathrm{I}}(Z_{\mathrm{n}})$ disebut graf ideal pembagi nol dengan himpunan titik yang sama dengan $AG_{\mathrm{I}}(Z_{\mathrm{n}})$ tetapi dua titik berbeda x dan y bertetangga jika dan hanya jika $xy \in I$, merupakan subgraf dari $AG_{\mathrm{I}}(Z_{\mathrm{n}})$. $AG_{\mathrm{I}}(Z_{\mathrm{n}})$ mempunyai sifat yaitu misal $R = Z_{\mathrm{n}}$ dan $I = < pq >$ maka $AG_{\mathrm{I}}(Z_{\mathrm{n}}) = \Gamma_{\mathrm{I}}\ (Z_{\mathrm{n}})$. Selain itu misalkan $R = Z_{\mathrm{n}}$ dan $I = < k >$, dimana $k \in Z_{\mathrm{n}}$. Jika ada titik $a, b \in \mathrm{V}(AG_{\mathrm{I}}(Z_{\mathrm{n}}))$ sedemikian rupa sehingga $\gcd\ (a, k) = \gcd\ (b, k)$ maka $\deg\ (a) = \deg\ (b)$ dan $N(a) \setminus \{b\} = N(b) \setminus \{a\}$. Dan misalkan Z_{n} adalah ring komutatif dan $I = < p^2 >$, dimana $p \in Z_{\mathrm{n}}$ maka $\mathrm{diam}\left(AG_{\mathrm{I}}(Z_{\mathrm{n}})\right) = 1$ dan $\mathrm{gr}\left(AG_{\mathrm{I}}(Z_{\mathrm{n}})\right) = 3$.

Kata Kunci : Graf annihilator ideal, Graf ideal pembagi nol, ring komutatif, ring bilangan bulat modulo *n*, ketetanggaan, nilai girth.

ABSTRACT

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Tittle : Characterization Ideal Annihilator Graph of Ring $\boldsymbol{Z}_{\mathrm{n}}$

Let R be a commutative ring with nonzero identity and I be a proper ideal of R, the annihilator graph of R with respect to I which is denoted by $AG_{\mathrm{I}}(R)$ is the undirected graph with vertex-set $V\big(AG_{\mathrm{I}}(R)\big) = \{x \in R - I;\ xy \in I \text{ untuk suatu } y \in R - I\}$ and two distinct vertices x and y are adjacent if and only if $A_{\mathrm{I}}(xy) \neq A_{\mathrm{I}}(x) \cup A_{\mathrm{I}}(y)$, where $A_{\mathrm{I}}(x) = \{r \in R\colon rx \in I\}$. Then $AG_{\mathrm{I}}(R)$ was developed with a special ring Z_{n} where Z_{n} is a commutative ring of integers modulo n with respect to I which is denoted by $AG_{\mathrm{I}}(Z_{\mathrm{n}})$ is the undirected graph with vertex-set $V\big(AG_{\mathrm{I}}(Z_{\mathrm{n}})\big) = \{x \in Z_{\mathrm{n}} - I;\ xy \in I \text{ untuk suatu } y \in Z_{\mathrm{n}} - I\}$, and two distinct vertices x and y are adjacent if and only if $A_{\mathrm{I}}(xy) \neq A_{\mathrm{I}}(x) \cup A_{\mathrm{I}}(y)$, where $A_{\mathrm{I}}(x) = \{r \in Z_{\mathrm{n}}\colon rx \in I\}$ for a $x \in Z_{\mathrm{n}}$. The undirected graph $\Gamma_{\mathrm{I}}(Z_{\mathrm{n}})$ is called zero divisor graph of R with respect to I with the same vertex-set of points as $AG_{\mathrm{I}}(Z_{\mathrm{n}})$ and two distinct vertices x and y are adjacent if and only if $xy \in I$, and is a subgraph of $AG_{\mathrm{I}}(Z_{\mathrm{n}})$. $AG_{\mathrm{I}}(Z_{\mathrm{n}})$ has basic properties is Let $R = Z_{\mathrm{n}}$ and $I = < pq >$, then $AG_{\mathrm{I}}(Z_{\mathrm{n}}) = \Gamma_{\mathrm{I}}\,(Z_{\mathrm{n}})$. In addition, suppose that $R = Z_{\mathrm{n}}$ dan $I = < k >$, where $k \in Z_{\mathrm{n}}$. If there exist vertices $a, b \in \mathrm{V}(AG_{\mathrm{I}}(Z_{\mathrm{n}}))$ such that gcd $(a, k) =$ gcd (b, k) then deg $(a) =$ deg (b) and N(a) \ {b} = N(b) \ {a}. and Let Z_{n} be a commutative ring and $I = < p^2 >$, where $p \in Z_{\mathrm{n}}$ then $\mathrm{diam}\big(AG_{\mathrm{I}}(Z_{\mathrm{n}})\big) = 1$ and $\mathrm{gr}\big(AG_{\mathrm{I}}(Z_{\mathrm{n}})\big) = 3$.

Keywords : Annihilator ideal graph, Ideal graph of zero divisor, Commutative ring, integers modulo *n* of ring, neighbourhood, girth.