the optimal solution of transportation problem

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THE OPTIMAL SOLUTION OF TRANSPORTATION PROBLEM USING INTERVAL POINT METHOD WITH PRODUCTION AND NON PRODUCTION COST CONSTRAINT

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Abstract

In an industry of a company or higher education institution must experience transportation problems. This transportation problem is required to schedule a delivery of goods with the objective to minimize transportation costs. Especially, the transportation problem in industry or Islamic Higher Education is to schedule the academic affairs. The objective of the transportation model is to plan the delivery of something from the source of the destination in such way as to minimize the total transportation cost. There are several cost constraints that occur in this transportation one of which is a budget constraint in which there are constraints of production and non-production costs.In transportation it is said to be unbalanced if the total number between source and destination is not the same. In this research there are methods to solve the unbalanced transportation problem with the production and non-production cost constraint using the Interval-Point method. This method is used to find the optimal solution.

Keywords: Production Cost, Non production cost, The Interval Point Method, Optimal Solution.

1. Introduction

In today's highly competitive market, the pressure on organizations to find better ways to create and deliver value to customers become stronger [1]. One of the studies in the optimization problem is optimization of transportation problems. Transportation problems are the process of placing resources in a particular location. The resolution of transportation problems in previous research using the transportation method. There are several transportation methods such as the North West Corner rule, the Least Cost rule, and the Vogel's Approximation Method (VAM) method [2].

Operations Research is the stages, methods and tools in an operation to obtain optimal results. The optimal results themselves are obtained using optimization which is part of operational research [3]. Optimization is a normative approach by identifying the best solution to a problem that is directed at the maximum and minimum points of an objective function. In accordance with its objectives, optimization can be developed in solving an application problem in the company or production or in Islamic Higher Education [4]. The basic transportation problem deals with the transportation problem of goods or services from a set of supply point to set of demand points so as to minimise linear transportation costs [5].

The objectives of this paper is to determine the optimal solution of the transportation problem in the Islamic Higher Education. There is one new method, namely the Interval-Point method. In this research, the authors completed four cases in academic affairs of Islamic Higher Education with different matrix sizes.

2. Methods

2.1. Unbalanced Transportation Problems with Budget Constraint

An interval transportation problem construct the data of supply, demand, and objective functions such as cost and time in some intervals [6]. Mathematically the unbalanced transportation problem with budget constraints can be written as follows: [7]

(P) Find value x_{ij}, where i = 1, 2, ..., m and j = 1, 2, ..., n. With condition as follows:

$$\sum_{j=1}^{n} x_{ij} \leq a_i, \quad i = 1, 2, \dots, m \quad (1)$$

$$\sum_{j=1}^{n} x_{ij} \in [b_j^1, b_j^2], \quad j = 1, 2, \dots, n \quad (2)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \in [z_1, z_2] \quad (3)$$

$$x_{ij} \geq 0, \quad i = 1, 2, \dots, m \; dan \; j = 1, 2, \dots, n \quad (4)$$

where,

c_{ij} : delivery cost from supply i to demand j

a_i : total demand

$[b_j^1, b_j^2]$: demand that not exactly at demand j

x_{ij} : total unit that deliver from supply i – th to demand j - th

$[z_1, z_2]$: not exactly budget

Unbalanced transportation that corresponding to (P) problems, given as follows:

(IP) Minimum $[z_1, z_2] = [\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}, \; \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}]$

Subject to:

$[\sum_{j=1}^{n} x_{ij}, \sum_{j=1}^{n} x_{ij}] = [a_i, a_i]$, i = 1, 2, ..., m

$[\sum_{i=1}^{m} x_{ij}, \sum_{i=1}^{m} x_{ij}] = [b_j^1, b_j^1]$, j = 1, 2, ..., n

$x_{ij} \geq 0, \; i = 1, 2, \dots, m \; and \; j = 1, 2, \dots, n$ and integer.

Now, we need theorem to find optimal solution from upper limit and lower limit, given as follows:

Theorem 1 If $\{y_{ij}^{\circ}$, for all i and j} is optimal solution for upper lower of transportation problem (UP) from (IP) problem, where [8]

(UP) Minimum $z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to:

$\sum_{j=1}^{n} x_{ij} = a_i$, i = 1, 2, ..., m

$\sum_{i=1}^{m} x_{ij} = b_j^2$, j = 1, 2, ..., n

$x_{ij} \geq 0$, i = 1, 2, ..., m and j = 1, 2, ..., n and integer

And if $\{x_{ij}^{\circ}$, for all i and j} is optimal solution from lower limit of transportation problem (LP) from (IP) problem, where[9]

(LP) Minimum $z_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to:

$\sum_{j=1}^{n} x_{ij} = a_i$, $i = 1, 2, \ldots$, m

$\sum_{i=1}^{m} x_{ij} = b_j^1$, $j = 1, 2, \ldots$, n

$x_{ij} \geq 0$, $i = 1, 2, \ldots$, m and $j = 1, 2, \ldots$, n and integer

then, for interval $\{[x_{ij}^{\circ}, y_{ij}^{\circ}], for\ all\ i$ and $j\}$ is optimal solution from (IP) problem subject to $x_{ij} \leq y_{ij}^{\circ}$, for all i and j.

2.2. Interval-Point Method

In 2012, D. Anuradha, P. Pandian, and G. Natarajan in his journal entitled "Solving Unbalanced Transportation Problems with Budgetary Constraints" introduced a new method for solving unbalanced transportation problems with budget constraints where supply was less than inventory. [10]

The stages in the Interval-Point method are as follows:

Stage 1: Form unbalanced interval transportation problems (IP) related to unbalanced transportation problems with budget constraints.

Stage 2: Form upper limit of unbalanced transportation problem (UP) from (IP) problem and solve the problem using zero point method. Assume $\{y_{ij}^{\circ}$, for all i and j} to be optimal solution from (UP) problem.

Stage 3: Form lower limit of unbalanced transportation problem (LP) from (IP) problem and solve the problem subject to $x_{ij}^{\circ} \leq y_{ij}^{\circ}$, for all i and j using zero point method. Assume $\{x_{ij}^{\circ}$, for all i and j} is optimal solution for (LP) problem with $x_{ij}^{\circ} \leq y_{ij}^{\circ}$, for all i and j.

Stage 4: Optimal solution for (LP) problem is $\{[x_{ij}^{\circ}, y_{ij}^{\circ}]$, for all i and $j\}$ that can be seen at Theorem 1. The optimal objective value for (IP) problem is $[Z_L, Z_U]$.

Stage 5: Assume $A = A_1 + A_2$ where A_1 is production constraint, A_2 is non production constraint with $A \in [Z_L, Z_U]$ for transportation cost of interval transportation problem, where A is budget cost constraint that can be written in the form of $A = Z_L + (Z_U - Z_L)\mu$, for $0 \leq \mu \leq 1$. Then $\mu = \frac{A - Z_L}{(Z_U - Z_L)}$.

Stage 6: Calculate the value of the decision variable using the formula: $x_{ij} = [x_{ij}^{\circ}, y_{ij}^{\circ}] = x_{ij}^{\circ} + (y_{ij}^{\circ} - x_{ij}^{\circ})\mu$ where μ value is got from Stage 5.

Stage 7: Optimal solution for UTPBC problem is $x_{ij} = x_{ij}^{\circ} + (y_{ij}^{\circ} - x_{ij}^{\circ})\left(\frac{A - Z_L}{(Z_U - Z_L)}\right)$ for given budget is A.

3. Results and Discussion

3.1. Case Studies

1. Case Study 1: Academic expenses transportation matrix shows in Table 1 [11].

Table 1. Transportation matrix (in thousand rupiahs)

	D_1	D_2	D_3	D_4	Supply
O_1	[2,2]	[9,9]	[3,3]	[7,7]	[12,16]
O_2	[7,7]	[4,4]	[5,5]	[4,4]	[9,10]
O_3	[2,2]	[6,6]	[9,9]	[2,2]	[10,10]
O_4	[4,4]	[6,6]	[6,6]	[7,7]	[6,6]
Demand	[12,12]	[7,8]	[7,7]	[6,6]	

Determine the costs incurred by the Islamic Higher Education with a production budget of 50,000 and a non-production budget of 50,000.

2. Case Study 2: Transportation matrix shows in Table 2.

Table 2. Transportation matrix

	S_1	S_2	S_3	S_4	S_5	Supply
A_1	[4,4]	[5,5]	[2,2]	[5,5]	[1,1]	[12,13]
A_2	[2,2]	[3,3]	[5,5]	[4,4]	[2,2]	[17,17]
A_3	[3,3]	[4,4]	[6,6]	[2,2]	[1,1]	[4,10]
A_4	[2,2]	[3,3]	[7,7]	[2,2]	[4,4]	[14,17]
Demand	[8,9]	[9,12]	[8,9]	[8,10]	[10,12]	

Determine the optimal costs incurred by the Islamic Higher Education to transport academic stuff from the source point to the destination with a production budget of 40 and a non-production budget of 50 (in thousand rupiahs).

3. Case Study 3: Transportation matrix shows in Table 3.

Table 3. Transportation matrix for the Islamic Higher Education

	S_1	S_2	S_3	S_4	Supply
B_1	[5,5]	[4,4]	[3,3]	[4,4]	[6,8]
B_2	[7,7]	[2,2]	[5,5]	[7,7]	[10,12]
B_3	[2,2]	[3,3]	[7,7]	[4,4]	[8,10]
B_4	[4,4]	[5,5]	[3,3]	[4,4]	[6,9]
B_5	[2,2]	[2,2]	[4,4]	[7,7]	[2,2]
Demand	[4,12]	[8,10]	[6,10]	[8,8]	

Determine the optimal cost incurred by the Islamic Higher Education.

4. Case Study 4: Transportation matrix shows in Table 4.

Table 4. Transportation matrix for the Islamic Higher Education

	D_1	D_2	D_3	D_4	D_5	Supply
C_1	[2,3]	[3,4]	[1,2]	[4,5]	[2,3]	[10,11]
C_2	[1,2]	[1,2]	[2,4]	[3,4]	[3,5]	[9,12]
C_3	[2,3]	[2,3]	[1,2]	[2,3]	[3,4]	[7,10]
C_4	[3,4]	[1,2]	[2,3]	[5,5]	[2,3]	[9,10]
C_5	[1,2]	[3,6]	[5,5]	[2,3]	[4,5]	[12,13]
Demand	[11,13]	[8,8]	[8,10]	[8,10]	[11,14]	

Determine the optimal cost incurred by the Islamic Higher Education.

3.2. Data Analysis

Transportation problem for Case Study 1 can be solved as follows:
Create interval transportation problem table as shows in Table 5.

Table 5. Interval transportation problem table

	D_1	D_2	D_3	D_4	Supply
O_1	[2,2]	[9,9]	[3,3]	[7,7]	[12,16]
O_2	[7,7]	[4,4]	[5,5]	[4,4]	[9,10]
O_3	[2,2]	[6,6]	[9,9]	[2,2]	[10,10]
O_4	[4,4]	[6,6]	[6,6]	[7,7]	[6,6]
Demand	[12,12]	[7,8]	[7,7]	[6,6]	

Next, create an upper bound table of interval problem.

Table 6. Upper bound table of interval problem

	D_1	D_2	D_3	D_4	Supply
O_1	2	9	3	7	16
O_2	7	4	5	4	10
O_3	2	6	9	2	10
O_4	4	6	6	7	6
Demand	12	8	7	6	

Use the zero point method to find the optimal solution to the upper bound problem. After using the zero point method, the optimal solution is obtained from the upper bound as follows:

$$\mathring{y}_{11} = 1,\ \mathring{y}_{13} = 6,\ \mathring{y}_{22} = 8,\ \mathring{y}_{23} = 1,\ \mathring{y}_{24} = 1,\ \mathring{y}_{31} = 5,\ \mathring{y}_{34} = 5,\ \mathring{y}_{41} = 6,\ \mathring{y}_{42} = 5$$

For next stage, create a lower bound table of interval problem then use the zero point method.

Table 7. Lower bound of interval problem table

	D_1	D_2	D_3	D_4	Supply
O_1	2	9	3	7	12
O_2	7	4	5	4	9
O_3	2	6	9	2	10
O_4	4	6	6	7	6

Demand	12	7	7	6	

After using the zero point method, the optimal solution is obtained from the lower bound as follows:

$$x^{\circ}_{11} = 1, x^{\circ}_{13} = 6, x^{\circ}_{22} = 7, x^{\circ}_{23} = 1 x^{\circ}_{24} = 1, x^{\circ}_{31} = 5, x^{\circ}_{34} = 5, x^{\circ}_{41} = 6$$

Because $x^{\circ}_{ij} \leq y^{\circ}_{ij}$, then the optimal solution to the problem (IP) is obtained $[x^{\circ}_{11}, y^{\circ}_{11}] = [1,1]$, $[x^{\circ}_{13}, y^{\circ}_{13}] = [6,6]$, $[x^{\circ}_{22}, y^{\circ}_{22}] = [7,8]$, $[x^{\circ}_{23}, y^{\circ}_{23}] = [1,1]$, $[x^{\circ}_{24}, y^{\circ}_{24}] = [1,1]$, $[x^{\circ}_{31}, y^{\circ}_{31}] = [5,5]$, $[x^{\circ}_{34}, y^{\circ}_{34}] = [5,5]$, dan $[x^{\circ}_{41}, y^{\circ}_{41}] = [6,6]$. Determine minimum interval transportation using Theorem 1, that is:

a For lower bound (Z_L) or $z_1 = 2(1) + 3(6) + 4(7) + 5(1) + 4(1) + 2(5) + 2(5) + 4(6) = 101$. As shows in Table 8.

b For upper bound (Z_U) or $z_2 = 2(1) + 3(6) + 4(8) + 5(1) + 4(1) + 2(5) + 2(5) + 4(6) = 105$. As shows in Table 9.

Thus, minimum interval transportation is $[101,105]$.

Table 8. Allocated lower bound table

	D_1	D_2	D_3	D_4	Supply
O_1	[2] **1**	[9]	[3] **6**	[7]	12
O_2	[7]	[4] **7**	[5] **1**	[4] **1**	9
O_3	[2] **5**	[6]	[9]	[2] **5**	10
O_4	[4] **6**	[6]	[6]	[7]	6
Demand	12	7	7	6	

Table 9. Allocated upper bound table

	D_1	D_2	D_3	D_4	Supply
O_1	[2] **1**	[9]	[3] **6**	[7]	16
O_2	[7]	[4] **8**	[5] **1**	[4] **1**	10
O_3	[2] **5**	[6]	[9]	[2] **5**	10
	[4]	[6]	[6]	[7]	6

O_4	**6**			
Demand	12	8	7	6

Transportation cost total $A = 101 + 4\mu$ where A is budget cost. Next, determine μ using $\mu = \frac{A-101}{4}$.

For next stage, calculate the value of each decision variable using $x_{ij} = [\mathring{x}_{ij}, \mathring{y}_{ij}] = \mathring{x}_{ij} + (\mathring{y}_{ij} - \mathring{x}_{ij})\mu$ that is: $x_{11} = 1$, $x_{13} = 5$, $x_{22} = 8 + \left(\frac{A-101}{4}\right)$, $x_{23} = 1$, $x_{24} = 1$, $x_{31} = 5$, $x_{34} = 7$, $x_{41} = 4$. Because $A = A_1 + A_2$ and $A_1 = 50$ meanwhile $A_2 = 50$ then A = 100. Thus, the optimal solution of unbalanced transportation problem with budget constraint are A = 100 is $x_{11} = 1$,$x_{13} = 5$,$x_{22} = 8{,}25$,$x_{23} = 1$, $x_{24} = 1$,$x_{31} = 5$, $x_{34} = 7$, $x_{41} = 4$. The cost incurred by the Islamic Higher Education is (1+5+ 8,25 + 1 + 1 + 5+7+4) = 32.25 (100000) or 3,225,000 rupiahs.

With the similar way, we obtain the solutions for Case study 2 – 4. From the case studies that the authors have done, the optimal solution to the problem of transportation using the interval-point method of different data sizes is as follows:

Table 10. The Optimal Solution Comparison

Case Study	Optimal Solution (Total Cost for Company in IDR)
Case Study 1: 4x4 matrix	3,225,000
Case Study 2: 4x5 matrix	4,382,000
Case Study 3: 5x4 matrix	354,400
Case Study 4: 5x5 matrix	466,300

3.Conclusions

To determine the optimal solution of interval transportation problems with budget constraints using the interval point method, namely the first stage, form a transportation table for the upper and lower limits, then use the zero point method to find the optimal solution from the upper and lower limits. After obtaining the optimal solution from the upper and lower limits, the optimal solution is obtained from the interval transportation problem. Then, find the minimum interval transportation using the Theorem 1. The next stage, calculate the values of each decision variable using the formula $x_{ij} = [\mathring{x}_{ij}, \mathring{y}_{ij}] = \mathring{x}_{ij} + (\mathring{y}_{ij} - \mathring{x}_{ij})\mu$. After that, calculate the budget cost constraints by adding up the production costs and non-production costs. Then get the minimum costs incurred by the Islamic Higher Education. For further research the services sector in industry or the Islamic Higher Education shall be more elaborated.

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