The Analysis Of Unbalanced Assignment Problems Using The Kotwal-Dhope Method To Develop A Massive Open Online Course

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***Abstract*— This study discusses the optimal solution of the unbalanced assignment of minimization case in order to develop massive open online course in the Islamic Higher Education by using a new method that is the Kotwal-Dhope Method. The method was formed with the help of the Hungarian method and the Matrix One's Assignment method was resolves the unbalanced assignment problem with a data size 8x4 which aims to minimize the total costs incurred by an Islamic Higher Education. The optimal solution with the Kotwal-Dhope Method begins by adding a dummy of one, carrying out the division operation of each column with the smallest element, after each row and column has a value of one, perform assignments in condition one so that each lecturer has their own job in develop massive open online course. Based on the results of this study it was found that, lecturer B was assigned to do stage 1 (Defining educational content), lecturer D was assigned to do stage 2 (Production and technical integration), lecturer E was assigned to do stage 3 (Communication), and lecturer H was assigned to do stage 4 (Course Animation and Overview). Then the assignments were: B → 1, D → 2, E → 3, H → 4. From the results of the assignment, the optimal solution for the minimum cost is 18 + 23 + 12 + 20 = 73 unit costs.**

Keywords*— a*ssignment problems; Kotwal-Dhope method; Massive Open Online Course; optimal solution.

I. Introduction

Mathematics is a science that has an important role in life. Mathematics acts as a way to solve various problems that occur in everyday life. In solving a problem, a mathematical model is usually used, one of which is operations research. Operations research (operation research) is the application of scientific methods to solve problems that arise in the implementation of activities so that the use of resources can be optimal and efficient. Operations research is a branch of science that has developed since World War II. At that time this method was only used in military activities, but subsequently this method was used in other fields, especially in industry, business and government administration [1, 2, 3].

Common problems solved using operations research include resource allocation, networking, inventory control, decision analysis, transportation and assignments [4, 5, 6, 7]. The assignment problem is a problem related to limited resources that must be distributed to various objectives, activities, and needs [8, 9, 10, 11]. This problem can be analogous to the problem of installing m jobs to n workers so that the total cost of the assignment required is as minimal as possible [12].

In general the assignment problem can be modeled as a linear program model with one objective function, namely the total cost of assigning each worker to each job [13, 14]. Currently, these assignment problems have been developed into multi-purpose assignment problems. The objective function to be optimized from this assignment model is not only related to the total cost of the assignment, but also optimizes revenue, mileage or production time [12].

The assignment problem is divided into two types, namely the balanced assignment problem and the unbalanced assignment problem. The method commonly used to solve the assignment problem is the Hungarian Method, because in solving the problem it can produce optimal solutions [15, 16, 17].

Along with the times, many new methods were created to solve assignment problems. One of the new methods is the Kotwal-Dhope method with a systematic procedure, easy to apply, and can produce optimal solutions. Where the Kotwal-Dhope Method is a combination of the Hungarian method and the Matrix One's Assignment (MOA) method [18].

Education, previously thought to be a bastion of tradition, has lately experienced dramatic changes through the incorporation of digital technology. Among these changes has been the introduction of MOOCs, massive, open, online courses that aim to provide a comprehensive educational format. In 2011, MOOCs reflected significant developing trends in education were introduced by several organizations such as Coursera, Udacity, and EDX. As known the role of higher educational institutions is to provide an individual with the skills and the knowledge. And to promote the idea of life-long learning. To achieve that higher education has been influenced by the rapid development of information and communication technologies. It has contributed to the creation of new technological means, such as MOOCs, to introduce the skills and techniques of learning observed in modern scientific research, thus allowing learners to gain a foothold in the competitive world [19]. In writing this paper the authors are interested in examining the problem of unbalanced assignments in create massive open online course in Islamic Higher Education with the Kotwal-Dhope method.

II. METHOD

The Kotwal-Dhope method is a new method designed to make it easier to solve the problem of minimizing unbalanced assignments. This method is used to provide the right assignment and minimize the costs incurred by the Islamic Higher Education [18].

The theoretical framework of the assignment problem with the Kotwal-Dhope Method can be expressed in the form of the c_{ij} matrix, assuming the costs associated with employee *i* are assigned to job *j* so that it is denoted c_{ij}. Then the matrix is as follows:

TABLE 1 KOTWAL-DHOPE METHOD ASSIGNMENT MATRIX

Employee	**Job**			
	1	***2***	**…**	***n***
1	c_{11} y_{11}	c_{12} y_{12}	…	c_{1n} y_{1n}
2	c_{21} y_{21}	c_{22} y_{22}	…	c_{2n} y_{2n}
⋮	⋮	⋮	⋮	⋮
M	c_{m1} y_{m1}	c_{m2} y_{m2}	…	c_{mn} y_{mn}

The assignment problem with the Kotwal-Dhope Method can be expressed in linear programming as follows [20, 21, 22, 23]:

$$\text{Min z} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} y_{ij}$$

Object to:

$$\sum_{j=1}^{n} y_{ij} = 1 \qquad i = 1,2,3,\dots,n$$

$$\sum_{i=1}^{n} y_{ij} = 1 \qquad j = 1,2,3,\dots,n$$

For y_{ij} = 0 or 1

Explanation:
z: assignment cost total
c_{ij}: assignment cost *i*th assignment for *j*th employee
y_{ij}: decision variable *i*th assignment for *j*th employee
i: employee index, *i = 1,2, …, n*
j: assignment index, *j = 1,2, …, m*

III. RESULT AND DISCUSSION

The object of this research focuses on cases of unbalanced assignment problems. The matrix of this unbalanced assignment problem has the order 8x4, which the case will be solved using the Kotwal-Dhope method. The purpose of this method is to get the best minimum solution results from the minimization problem on the unbalanced assignment problem.

The Islamic Higher Education wants to develop a massive open online course (MOOC). There are 4 stage to creating MOOC, namely defining educational content, production and technical integration, communication and course animation and overview. The head of department offers 4 different stage positions to 8 lecturers. The Islamic Higher Education wants to determine the right lecturer for each stage with various predetermined criteria. The cost for each stage has been determined (in unit cost). The matrix in Table 2 shows the employee costs for each different stage.

TABLE 2 ASSIGNMENT COST MATRIX

Lecturer	**Stage**			
	1	***2***	***3***	***4***
A	53	62	42	89
B	18	35	39	55
C	93	80	91	83
D	79	23	96	56
E	43	16	12	20
F	87	70	87	31
G	35	79	25	59
H	27	16	12	20

Stage 1: defining educational content; Stage 2: production and technical integration; Stage 3: communication; Stage 4: course animation and overview

Step 1: Because the given matrix is an unbalanced matrix, balance the matrix by adding a dummy with a value of 1 in the work column. Can be seen in Table 3.

TABLE 3 MATRIKS BALANCED

Lecturer	Stage							
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***
A	53	62	42	89	1	1	1	1
B	18	35	39	55	1	1	1	1
C	93	80	91	83	1	1	1	1
D	79	23	96	56	1	1	1	1
E	43	16	12	20	1	1	1	1
F	87	70	87	31	1	1	1	1
G	35	79	25	59	1	1	1	1
H	27	16	12	20	1	1	1	1

Step 2: Select and mark the minimum value of each column (Table 4). Then perform the division operation on each value in the column with the smallest value that has been selected. The results are shown in Table 5.

TABLE 3 MINIMUM VALUE IN EACH COLUMN

Lecturer	Stage							
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***
A	53	62	42	89	1	1	1	1
B	18	35	39	55	1	1	1	1
C	93	80	91	83	1	1	1	1
D	79	23	96	56	1	1	1	1
E	43	16	12	20	1	1	1	1
F	87	70	87	31	1	1	1	1
G	35	79	25	59	1	1	1	1
H	27	16	12	20	1	1	1	1

= Cell wirh minimum value in column

Tabel 3. 4 Hasil Operasi Pembagian Setiap Kolom

Lecturer	Stage							
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***
A	2.94	3.87	3.5	4.45	1	1	1	1
B	1	2.18	3.25	2.75	1	1	1	1
C	5.16	5	7.58	4.15	1	1	1	1
D	4.38	1.43	8	2.8	1	1	1	1
E	2.38	1	1	1	1	1	1	1
F	4.83	4.37	7.25	1.55	1	1	1	1
G	1.94	4.93	2.08	2.95	1	1	1	1
H	1.5	1	1	1	1	1	1	1

Step 3: Draw as minimum lines as possible on the rows and columns to cover the value 1 (Table 6). If the number of lines (ℓ) = n then the optimal solution is found and go to step 6. However, if (ℓ) n then go to step 5.

TABLE 5 DRAWING LINE VALUE 1 IN FIRST ITERARATION

Lecturer	Stage							
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***
A	2.94	3.87	3.5	4.45	1	1	1	1
B	1	2.18	3.25	2.75	1	1	1	1
C	5.16	5	7.58	4.15	1	1	1	1
D	4.38	1.43	8	2.8	1	1	1	1
E	2.38	1	1	1	1	1	1	1
F	4.83	4.37	7.25	1.55	1	1	1	1
G	1.94	4.93	2.08	2.95	1	1	1	1
H	1.5	1	1	1	1	1	1	1

= Line value 1

Step 4: Since the number of lines (ℓ) is n, select and mark the minimum value that is not affected by the line (Table 7). Perform the division operation on the value that is not affected by the line with the minimum value that has been selected (Table 8), then replace the value that is hit by the line twice with that minimum value (Table 9) and for those affected by the line the value is remain the same.

TABLE 6 MINIMUM VALUE IN FIRST ITERATION

Lecturer	Stage							
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***
A	2.94	3.87	3.5	4.45	1	1	1	1
B	1	2.18	3.25	2.75	1	1	1	1
C	5.16	5	7.58	4.15	1	1	1	1
D	4.38	1.43	8	2.8	1	1	1	1
E	2.38	1	1	1	1	1	1	1
F	4.83	4.37	7.25	1.55	1	1	1	1
G	1.94	4.93	2.08	2.95	1	1	1	1
H	1.5	1	1	1	1	1	1	1

TABLE 7 DIVIDE OPERATION RESULT IN FIRST ITERATION

Lecturer	**Stage**							
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***
A	2.94	2.70	2.44	3.11	1	1	1	1
B	1	1.52	2.27	1.92	1	1	1	1
C	5.16	3.49	5.3	2.9	1	1	1	1
D	4.38	1	5.59	1.95	1	1	1	1
E	2.38	1	1	1	1	1	1	1
F	4.83	3.05	5.06	1.08	1	1	1	1
G	1.94	3.44	1.45	2.06	1	1	1	1
H	1.5	1	1	1	1	1	1	1

TABLE 9 REPLACEMENT VALUES EXPOSED TO A LINE TWICE

Lecturer	**Stage**							
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***
A	2.94	2.70	2.44	3.11	1	1	1	1
B	1	1.52	2.27	1.92	1	1	1	1
C	5.16	3.49	5.3	2.9	1	1	1	1
D	4.38	1	5.59	1.95	1	1	1	1
E	1.43	1	1	1	1.43	1.43	1.43	1.43
F	4.83	3.05	5.06	1.08	1	1	1	1
G	1.94	3.44	1.45	2.06	1	1	1	1
H	1.43	1	1	1	1.43	1.43	1.43	1.43

Step 5: Draw minimal lines on the rows and columns to cover the value 1 (Table 3.10).

TABLE 10 DRAWING LINES CONTAINING VALUE 1 IN THE SECOND ITERATION

Lecturer	**Stage**							
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***
A	2.94	2.70	2.44	3.11	1	1	1	1
B	1	1.52	2.27	1.92	1	1	1	1
C	5.16	3.49	5.3	2.9	1	1	1	1
D	4.38	1	5.59	1.95	1	1	1	1
E	1.43	1	1	1	1.43	1.43	1.43	1.43
F	4.83	3.05	5.06	1.08	1	1	1	1
G	1.94	3.44	1.45	2.06	1	1	1	1
H	1.43	1	1	1	1.43	1.43	1.43	1.43

TABLE 11 ASSIGNMENT OF STAGE WITH DRAW A LINE

Lecturer	**Stage**							
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***
A	2.94	2.70	2.44	3.11	1	1	1	1
B	1	1.52	2.27	1.92	1	1	1	1
C	5.16	3.49	5.3	2.9	1	1	1	1
D	4.38	1	5.59	1.95	1	1	1	1
E	1.43	1	1	1	1.43	1.43	1.43	1.43
F	4.83	3.05	5.06	1.08	1	1	1	1
G	1.94	3.44	1.45	2.06	1	1	1	1
H	1.43	1	1	1	1.43	1.43	1.43	1.43

TABLE 8 KOTWAL-DHOPE METHOD ASSIGNMENT RESULT

Lecturer	**Stage**							
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***
A	53	62	42	89	1	1	1	1
B	18	35	39	55	1	1	1	1
C	93	80	91	83	1	1	1	1
D	79	23	96	56	1	1	1	1
E	43	16	12	20	1	1	1	1
F	87	70	87	31	1	1	1	1
G	35	79	25	59	1	1	1	1
H	27	16	12	20	1	1	1	1

: final result

Step 6: Find lecturer who are assigned to only do one stage on the row and column to find the optimal solution in Table 10. Since the number of lines (ℓ) = n, the assignment problem has obtained the optimal solution. Lecturer B is assigned to stage 1, lecturer D is assigned to stage 2, lecturer E is assigned to stage 3, and lecturer H is assigned to stage 4.

Thus, the optimal assignment schedule with a predetermined minimum cost is Lecturer A is not assigned to do stage, Lecturer B is assigned to do stag 1 for 18 unit costs, Lecturer C is not assigned to do stage, Lecturer D is assigned to do stage 2 for 23 unit costs, Lecturer E is assigned to do stage 3 for 12 unit costs, Lecturer F is not assigned to do stage, Lecturer G is not assigned to do stage, and Lecturer H is assigned to do stage 4 for 20 unit costs. With a minimum total cost, namely:

$$\text{Min } z = (18 \times 1) + (23 \times 1) + (12 \times 1) + (20 \times 1) = 73$$

Then the total minimum costs incurred by the Islamic Higher Education to develop a massive open online course is 73 unit costs. The Islamic Higher Education should have a plan to develop MOOC as a part of college teaching and learning with considering cost efficiency.

IV. CONCLUSION

From the results of the discussion and data analysis in this paper, the Kotwal-Dhope method is a new method designed to make it easier to solve the problem of unbalanced assignment in minimization cases. The Kotwal-Dhope method has a systematic procedure to facilitate the search for optimal solutions. There is need to experiment with the application of a curriculum in the Islamic Higher Education institution using Massive Open Online Course, and then evaluate this experience as a first step.

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