**The Online Learning Assignment Optimization Using Average Total Opportunity Cost**

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**Abstract.** In everyday life there are problems in decision making. The problem is model mathematically with Linear programming. One of the problems is allocating limited resources to obtain the maximum profit or minimize the cost of production. The assignment problem is one of method in making decisions by allocating m sources to n destinations optimally. This study aims to understand the application of the Average Total Opportunity Cost Method. The Average Total Opportunity Cost method is a method for solving the assignment problem by selecting the biggest average total cost between the Row Average Total Opportunity Cost and the Column Average Total Opportunity Cost.

# INTRODUCTION

The era of globalization has led to a rapid increase in science and technology so that it leads people to be fast and precise in making decisions. One of the branches of science and technology is mathematics. Mathematics has two branches, namely pure mathematics and applied mathematics. Pure mathematics represents abstract variables and their properties by definition. Applied mathematics represents variables that can be seen in real events. Mathematics that is widely used by most people is usually applied mathematics, because it can be applied in solving problems in everyday life. Applications of applied mathematics in everyday life, for example in decision making.

In the industrial sector there is a study of operations research which is a matter related to optimal decision making of various systems originating from real life, characterized by the need to allocate limited resources. The purpose of the allocation is to minimize losses and or maximize profits in the decision-making process that needs to be supported by careful calculations. Therefore, it is not enough to just consider the need for techniques, equipment or methods to solve problems in the industrial sector, one of which is the assignment problem.

Assignment problem is one way for the decision-making process. In the industrial sector as well as in higher education, we are often faced with several choices that require the optimal assignment of various productivity sources that have different levels of optimization in the hope of minimizing costs or maximizing profits.

Many new methods were born to solve assignment problems along with the times. A method is said to be new not only based on the year of publication but there are other factors, namely the method has just been developed by the author in solving the assignment problem. For example, the Average Total Opportunity Cost (ATOC) method was originally used to solve transportation problems but was only introduced by R Sophia Porchelvi and M. Anitha as a method for solving assignment problems.

**LITERATURE REVIEWS**

## Assignment Problem

## The assignment problem is a special case of the linear programming problem. In the industrial world as well as in Islamic Higher Education, management often faces problems related to the optimal assignment of various productive sources that have different levels of efficiency for different tasks. The assignment problem will cover a number of n sources that have n assignments [2].

## The term assignment implies that one person will do a certain task. The assignment model may be thought of as a special form of the transportation model. In this model, the capacity of the i-th source and j-th destination is 1 according to the basic concept, so that the optimal value of the decision variable must only be zero or one. Equations (2.3) and (2.4) will be simplified in the assignment model so that and must only be worth 1 so that the optimal value of the decision variable is only one or zero. Therefore, the general form of the assignment mathematical model is [4]:

subject to:

where ,

**TABLE 1.** Assignment Matrix

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| SOURCE | DESTINATION | | | | Source Capacity per Period |
|  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |
|  |  |  |  |  |  |
|  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |
| Destination Demand per Period |  |  |  |  |  |
|  |  |  |  |  |

## Average Total Opportunity Cost (ATOC) Method

The Average Total Opportunity Cost (ATOC) method is a new method for assignment problems, but this method is usually used to solve transportation problems by using the average total cost in each row and column. The assignment problem solving algorithm using the Average Total Opportunity Cost (ATOC) method has 2 parts, namely [8]:

A. Algorithm for creating a total cost table.

1. If the objective function of the assignment matrix is ​​to maximize, then identify the largest element in the assignment matrix and then subtract all the elements of the assignment matrix with the largest element. If the objective function of the assignment matrix is ​​for minimization, go to step 2.
2. If the data size of the assignment problem case is not balanced then add a dummy so that the data size is balanced.
3. Find the smallest element in each row. Subtract the smallest element by all the elements in each row of the assignment table and place the result at the top right of the corresponding element.
4. Find the smallest element in each column. Apply the same operation to each column and place the result at the bottom right of the appropriate element.
5. Form TOCT (Table of Opportunity Cost Total) whose entry value is the sum of the top right and bottom right elements from step 1 and step 2.

B. Algorithm for assignment allocation.

1. Calculate the average total opportunity cost (ATOC) in the cell of each row identified as RATOC (Row Average Total Opportunity Cost) and place it to the right of the objective column. And calculate the average total opportunity cost (ATOC) in the cell of each column identified as CATOC (Column Average Total Opportunity Cost) and place it under the source row.
2. Identify the highest element between RATOC and CATOC then select the smallest element from the row/column identified as having the highest element with the following conditions:
   1. If there are 2 or more highest elements, select the highest element which has the smallest cost.
   2. If there are 2 or more elements of the smallest cost, select one at random.
3. Allocate resources to appropriate destinations according to step 2. Reduce corresponding rows and columns after they are allocated.
4. Repeat step 1 to step 3 until the optimal solution is met.
5. Calculate the optimal solution.

# RESULT AND DISCUSSION

The simulation data in this study is data for assignment problems with unbalanced data where an Islamic Higher Education has 5 application programs for online learning to be developed. Islamic Higher Education is currently holding tests for 6 online learning developers. The time it takes for an online learning developer to run an online learning application program varies. The data in Table 2 shows the time (hours) of each online learning developer running their application program [16]. In this study, a minimum time will be determined to develop a learning application program at the Islamic Higher Education.

**TABLE 2.** Time Data for Each Online Learning Developer

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Program** | **Online Learning Developer** | | | | | |
|  |  |  |  |  |  |
|  | 80 | 140 | 80 | 100 | 56 | 98 |
|  | 48 | 64 | 48 | 126 | 170 | 100 |
|  | 56 | 80 | 120 | 100 | 70 | 64 |
|  | 99 | 100 | 100 | 104 | 80 | 90 |
|  | 64 | 90 | 90 | 60 | 60 | 70 |

1. Algorithm for make total cost matrix.

After applying Step 1 and 2 in ATOC method, then the results are represented in Table 3.

**TABLE 3.** Cost Assignment Data with Add Dummy

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Program** | **Online Learning Developer** | | | | | |
|  |  |  |  |  |  |
|  | 80 | 140 | 80 | 100 | 56 | 98 |
|  | 48 | 64 | 48 | 126 | 170 | 100 |
|  | 56 | 80 | 120 | 100 | 70 | 64 |
|  | 99 | 100 | 100 | 104 | 80 | 90 |
|  | 64 | 90 | 90 | 60 | 60 | 70 |
|  | 0 | 0 | 0 | 0 | 0 | 0 |

After applying Step 3 and 4 in ATOC method, then the results are represented in Table 4.

**TABLE 4.** Subtraction Result for Each Row and Column

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Program** | **Online Learning Developer** | | | | | |
|  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
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|  |  |  |  |  |  |  |

The TOCT (Table of Opportunity Cost Total) form whose entry value is the sum of the top right and bottom right elements from step 3 and 4.

**TABLE 5.** TOCT Form

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Program** | **Online Learning Developer** | | | | | |
|  |  |  |  |  |  |
|  | 104 | 224 | 104 | 144 | 56 | 140 |
|  | 48 | 80 | 48 | 204 | 292 | 152 |
|  | 56 | 104 | 184 | 144 | 84 | 72 |
|  | 118 | 120 | 120 | 128 | 80 | 100 |
|  | 68 | 120 | 120 | 60 | 60 | 80 |
|  | 0 | 0 | 0 | 0 | 0 | 0 |

1. Algoritm for assignment allocation.

From calculation results RATOC and CATOC in Step 1, then the results are represented in Table 6.

**TABLE 6.**  Calculation Results RATOC and CATOC at First Allocation

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Program** | **Online Learning Developer** | | | | | | **RATOC** |
|  |  |  |  |  |  |
|  | 104 | 224 | 104 | 144 | 56 | 140 | 128,67 |
|  | 48 | 80 | 48 | 204 | 292 | 152 | 137,33 |
|  | 56 | 104 | 184 | 144 | 84 | 72 | 107,33 |
|  | 118 | 120 | 120 | 128 | 80 | 100 | 111 |
|  | 68 | 120 | 120 | 60 | 60 | 80 | 84,67 |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **CATOC** | 65,6 | 108 | 96 | 113,3 | 95,3 | 90,6 |  |

After identify the biggest element at Step 2, the results are represented in Table 7.

**TABLE 7.** Identification Result of The Biggest Element in First Allocation

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Program** | **Online Learning Developer** | | | | | | **RATOC** |
|  |  |  |  |  |  |
|  | 104 | 224 | 104 | 144 | 56 | 140 | 128,67 |
|  | 48 | 80 | 48 | 204 | 292 | 152 | 137,3 |
|  | 56 | 104 | 184 | 144 | 84 | 72 | 107,3 |
|  | 118 | 120 | 120 | 128 | 80 | 100 | 111 |
|  | 68 | 120 | 120 | 60 | 60 | 80 | 84,6 |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **CATOC** | 65,6 | 108 | 98 | 113,3 | 95,3 | 90,6 |  |

The explanation:

: Cell with the biggest element between RATOC and CATOC

: Cell with the smallest that choose from row or column with the biggest average element

Based on the identification of the largest element in step 2, the first optimal allocation is obtained, namely the online learning developer working on the online learning application program .

**TABLE 8.** First Assignment Allocation Result

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Program** | **Online Learning Developer** | | | | | | **RATOC** |
|  |  |  |  |  |  |
|  | 104 | 224 | 104 | 144 | 56 | 140 | 112 |
|  | 48 | 80 | 48 | 204 | 292 | 152 | 139,3 |
|  | 56 | 104 | 184 | 144 | 84 | 72 | 107,3 |
|  | 118 | 120 | 120 | 128 | 80 | 100 | 111 |
|  | 68 | 120 | 120 | 60 | 60 | 80 | 84,6 |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **CATOC** | 65,6 | 108 | 98 | 113,3 | 95,3 | 90,6 |  |

The explanation:

: Cell of Allocation Result from Pogram *i* to Online Learning Developer *j*

**TABLE 9.**  Reduction Result Row and Column in First Allocation

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Program** | **Online Learning Developer** | | | | |
|  |  |  |  |  |
|  | 104 | 224 | 144 | 56 | 140 |
|  | 56 | 104 | 144 | 84 | 72 |
|  | 118 | 120 | 128 | 80 | 100 |
|  | 68 | 120 | 60 | 60 | 80 |
|  | 0 | 0 | 0 | 0 | 0 |

Repeat steps 1 to step 3 until the optimal solution is met. In this research up to six iterations. So that the assignment of each online learning development to each online learning application program is optimal as shown in Table 10.

**TABLE 10.** Final Result of Assignment Allocation

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Program** | **Online Learning Developer** | | | | | |
|  |  |  |  |  |  |
|  | 80 | 140 | 80 | 100 | 56 | 98 |
|  | 48 | 64 | 48 | 126 | 170 | 100 |
|  | 56 | 80 | 120 | 100 | 70 | 64 |
|  | 99 | 100 | 100 | 104 | 80 | 90 |
|  | 64 | 90 | 90 | 60 | 60 | 70 |
|  | 0 | 0 | 0 | 0 | 0 | 0 |

Based on Table 10, the optimal solution for the assignment problem is obtained with the following details: Program I was carried out by Online Learning Developer 5 with a minimum time of 56; Program II is done by Online Developer Learning Developer 3 with a minimum time of 48; Program III is carried out by Online Learning Developer 1 with a minimum time of 56; Program IV is done by Online Learning Developer 6 with a minimum time of 90; Program V is done by Online Learning Developer 2 with a minimum time of 60; because Program VI is a dummy program thus the minimum time on the assignment is zero.

So that the minimum total cost can be calculated:

So that the results obtained for the minimization case with unbalanced data using the Average Total Opportunity Cost (ATOC) method, the optimal time in working on the application in online learning is 310 hours.

# CONCLUSION

The application of the Average Total Opportunity Cost (ATOC) method in this study was used to solve the assignment problem of developing online learning in Islamic Higher Education. The Average Total Opportunity Cost (ATOC) method is a method for solving assignment problems by forming a Table of Total Opportunity Cost (TOCT) and then finding the largest average of the total costs in rows and columns to be allocated in each iteration. For further research, it will be applied to other assignment problems related to the learning process at Islamic Higher Education by increasing the size of the data and the process is carried out with programs to make it easier.

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